Velimir Khlebnikov's *Laws of Time*

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Abstract

In February 2019, the video artist Kristina Paustian honored the Russian futurist Velimir Khlebnikov with her solo exhibition *Laws of Time. The future calculations by Velimir Khlebnikov* in Berlin, Germany. Among other aspects, she focused on his poetry and the mathematics used in his works. In this paper, we rephrase Khlebnikov's ideas and thoughts in mathematical terms. Furthermore, we prove that his method of foretelling the future by analyzing several events which took place in the past produces only random results. Finally, we describe the corresponding mathematical part of the exhibition and provide an artistic exploration of Khlebnikov's methodology.

Velimir Khlebnikov

Several Russian authors and poets of the 20th century were interested in mathematics and used it to explain or illustrate a variety of events. In [5], the three authors Pavel Florenskij (1882–1937), Daniil Charms (1905–1942), and Velimir Khlebnikov were investigated due to their ways of dealing with mathematics in their writings. His contemporaries called Khlebnikov "Lobachevsky of words"[1]. His writings allude to a huge number of mathematical concepts from algebra, number theory and geometry. For example, he interprets $\sqrt{-1}$ not only as a (special) number, but literally as the root of a plant and—because of the Russian word for root—also a root of language. This creates a direct connection between poetry and mathematics ([5, p. 54]). In February 2019, the video artist Kristina Paustian honored Khlebnikov with her solo exhibition focusing on his dealing with Diophantine equations. The exhibition was titled *Laws of Time. The future calculations by Velimir Khlebnikov*.

Velimir Khlebnikov was born as Viktor Vladimirovich Khlebnikov in Malye Derbety, Russian Empire (in present day Kalmykia), in 1885. He attended school first in Kazan and later in Saint Petersburg. Khlebnikov was a member of *Hylaea*—a significant group of Russian futurists. During his studies he attended different courses including mathematics, natural sciences, and Slavic studies. His enthusiasm for mathematics is reflected in several of his works. He promoted his "destiny sciences": Using mathematical tools to relate several dates of historical events. In his *Tables of Destiny* (1915), he investigated aspects of mathematical powers. In particular, powers of 3 are interesting to him, as he related the numbers of days in a year to these powers by: $365 = 3^5 + 3^4 + 3^3 + 3^2 + 3^1 + 3^0 + 1$. Note that an additional 1 is necessary here, which shows Khlebnikov's willingness to sacrifice rigor in order to obtain the desired results. Unveiling these relations is to him like discovering the mysteries of the "Russian Atlantis", the sunken city of Kitezh. He wrote:

> *In the famous old legend, the city of Kitezh lay sunk in a deep dark lake in the forest, while here, out of each spot of time, out of every lake of time, arose an orderly multinomial of threes with towers and steeples, just like another Kitezh. [. . .] A city of threes with its towers and steeples rings loudly from out of the depths of time. An orderly city with numerical towers has replaced previous visions of spots of time.* [1, p. 420]

[1]"Lobačevskij des Wortes", see [5, p. 50].

The *Tables of Destiny* proceed to relate several historical events by writing the difference between their dates as sum of powers of 3. Khlebnikov's admiration for these constructions is probably best expressed in his poem [1, p. 430]:

> *The life of centuries in the light of* 3^n.
> *The eternal duel, illuminated by the torches of* 3^n.
> *The staff of victory changes hands, passed from one warrior to another.*
> *Waves of two worlds, the alternating spears of East and West, clashing through the centuries.*

To him, these numbers carry semantics as "the equation 3^n [is] an equation of death"[2]. Khlebnikov died in 1922 in Santalowo, Russia.

Teacher and Student

The piece *Teacher and Student* is part of Khlebnikov's theoretical writings on language [1]. The editors of his collected works characterize this part as follows: "*His intention is to create an international system of communication, to provide humanity with a single, universal, scientifically constructed language.*" [1, p. 265]. This is in accordance with the general scheme of the manifesto *A Slap in the Face of Public Taste* (1912), issued by a group of Russian futurists including Khlebnikov, which emphasized the need for poets to create a new language and to throw the classic authors overboard.

As illustrated above, Khlebnikov uses mathematical notions in his writing and poetry. In the conversation *Teacher and Student* (1912), he 'calculated' and predicted a fall of a state to happen in 1917. Later on, he took great pride in this prediction as he related it to the Russian Revolution. Specifically, Khlebnikov presents the following conversation:

> ***Teacher**: And what else have you discovered?*
> ***Student**: You see, I keep thinking about the action of the future on the past. But given the weight of ancient books that keeps pressing down on humanity, is it even possible to conceive such matters? No, mortal, cast your eyes peacefully downward! Whatever happened to the great destroyers of books? Their waves are as shaky a footing as the dry land of ignorance!*
> ***Teacher**: Anything else?*
> ***Student**: Anything else? Yes! You see, what I wanted was to read the writing traced by destiny on the scroll of human affairs. [. . .] I have discovered that in general a time period Z separates similar events:*
>
> $$Z = (365 + 48 \cdot y) \cdot x, \tag{1}$$
>
> *where y can have a positive or a negative value. [. . .] The conquest of Egypt in 1250 corresponds to the fall of the kingdom of Pergamum in 133. The Polovtsians overran the Russian steppe in 1093, 1383 years after the fall of Samnium in 290. And in 534 the kingdom of the Vandals was subjugated. Should we not therefore expect some state to fall in 1917?*

Here, Khlebnikov claims that the polynomial (1) relates two dates in history and thus can even be used to predict the future. Two dates are related if their difference can be represented by polynomial (1) for some integer values x and y. For $x = 3$, $y = 2$, the fall of Egypt in 1250 and the fall of Pergamum in -133 are related by

$$1250 - (-133) = 1383 = (365 + 48 \cdot 2) \cdot 3,$$

[2]"die Gleichung 3^n eine Gleichung des Todes" [5, p. 77]

while for the same x and y, the dates of 534 and 1917 are related by

$$1917 - 534 = 1383 = (365 + 48 \cdot 2) \cdot 3.$$

Note that not all differences can be realized via polynomial (1). For instance, there are no integers x and y such that $Z = 12$ (see Section "Expected Number of Connections" for further details).

Especially in his theoretical writings [1], Khlebnikov illustrates the ideas and results of his research with mathematical notions, such as sums of integer powers of natural numbers (see *Excerpt from The Tables of Destiny* in [1, p. 417]), or complex numbers to create the image of turning something inside-out [8, p. 253]. For Khlebnikov, the chosen numbers have semantics: "365 is the number of days of the year ('solar year'), [which] doubles as a natural constant"[3]. A similar meaning is given to another "constant" in polynomial (1): $48 = 4$ weeks $\cdot$ 12 month as the number of weeks in a year. Or—as Khlebnikov explains in a letter [5, p. 71]—it is $\sqrt{365} + 28 = 47.104\ldots \approx 48$. We can see here that Khlebnikov is willing to bend seemingly objective mathematical statements to fit his intentions. The following is devoted to further deconstruct his methodology.

Khlebnikov's Method from a Mathematical Viewpoint

The polynomial (1) spans a surface in $\mathbb{R}^3$, with integer points $(x, y, Z = (365 + 48 \cdot y) \cdot x)$ on it, see Figure 3. In total, Khlebnikov uses 77 dates in his conversation *Student and Teacher*. These are given as vertices of the graphs in Figures 1a and 1b. In the first of these two graphs, we draw a connection between two vertices a and b, if Khlebnikov established values x and y in his writings to connect their difference $Z = (a - b)$ via polynomial (1). Note that he also reported connections that differ by ± 1 from values obtained by polynomial (1) shown as dotted lines. In the second graph shown in Figure 1b, we draw some additional dashed lines. These indicate that a connection—not reported by Khlebnikov in his writings—between the given dates via polynomial (1) exists, when using those x and y values Khlebnikov used in other cases in his publication.

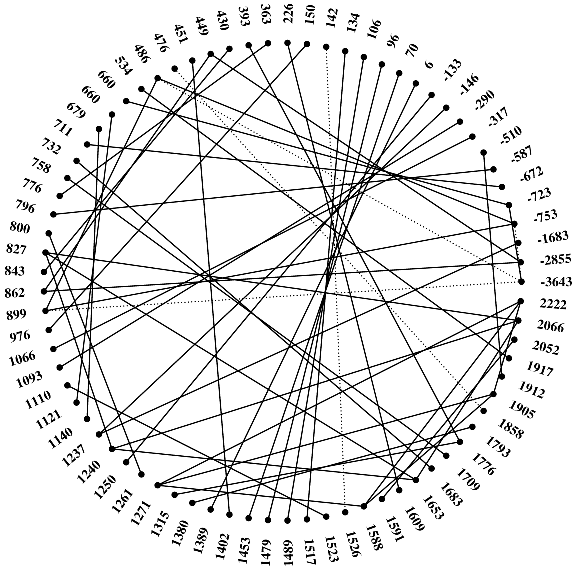
That is, Figures 1a and 1b show the connections between the dates established by Khlebnikov and those connections he could have found using already occuring y values and arbitrary x values in polynomial (1). Taking a step further and considering all possible values for x and y, there are many more connections in Khlebnikov's data, as shown in Figure 1c. Finally, the graph given in Figure 1d presents the same constructions as Figure 1c, also with 77 data points that are chosen randomly from a uniform distribution on $\{z \in \mathbb{Z} : -3000 \leq z \leq 2000\}$. This first experiment already provides an indication that the connections obtained from Velimir Khlebnikov are not significantly different from random connections.

To further quantify the relation between Khlebnikov's connections and connections expected from random data, we use the following notation

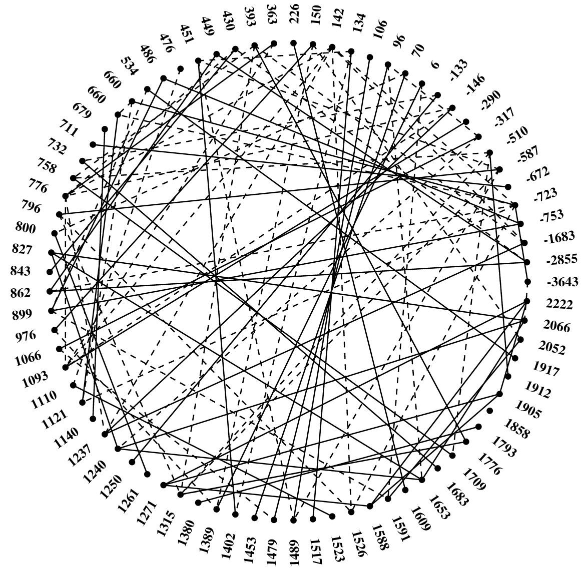
$$P(x, y) = (365 + 48y)x \tag{2}$$

with $x, y \in \mathbb{Z}$, for the polynomial introduced by Khlebnikov as given in polynomial (1). Furthermore, consider the set $\mathcal{I}_a := \{z \in \mathbb{Z} : 0 \leq z \leq a\}$, where $a \in \mathbb{N}$. Note here that we are only considering distances between points from the set $\mathcal{I}_a$, thus we can shift any such set to begin at zero. Therefore, in the following we will investigate $a = 5,000$ as inspired by Velimir Khlebnikov's writing. Pick numbers $m_1, \ldots, m_n \in \mathcal{I}_a$ randomly. These will represent Khlebnikov's dates. Two numbers m_i, m_j are called *connected* if there exist integers $x_{ij}, y_{ij} \in \mathbb{Z}$ such that $m_i - m_j = P(x_{ij}, y_{ij})$. Note that two numbers m_i, m_j, $i \neq j$, can be equal. These come from historical events and we do allow two events to occur in the same year. These are trivially connected via $x_{ij} = 0$. Now, the question of interest is: What is the expected number of connected pairs in the set $\{m_1, \ldots, m_n\}$?

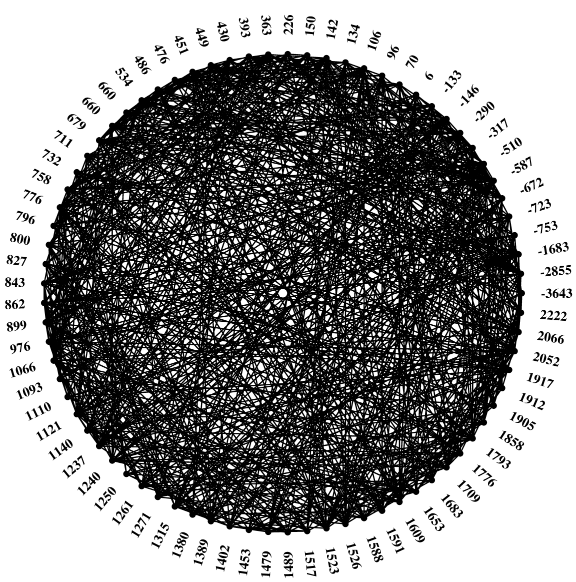
[3]"[...] 365, die als Zahl der Tage des Jahres ('Sonnenjahr') gleichsam eine Naturkonstante ist." [5, p. 70], cf. [5, pp. 70–73] for further use and interpretation of 365 in Khlebnikov's work.



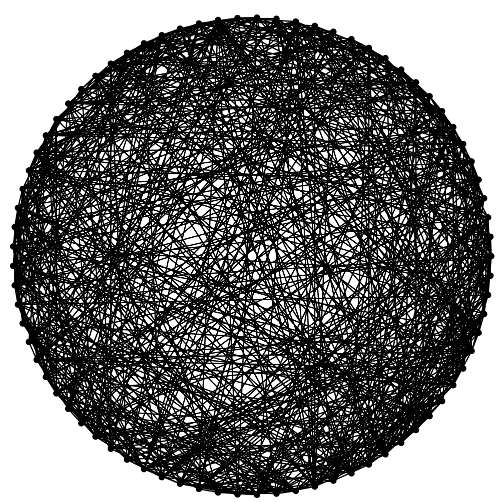
(a) *The dates used by Khlebnikov in his conversation* Teacher and Student *as vertices of a graph. An edge is drawn if Khlebnikov established a connection between the two dates. The dotted lines have differences that are* ±1 *away from values by polynomial (1).*



(b) *The same graph as in Figure 1a, but with all connections established by polynomial (1) when using the* y *values of Khlebnikov from his publication and arbitrary* x *values.*



(c) *All connections satisfying polynomial (1), with Klebnikov's dates and all possible values for* x *and* y*.*



(d) *All connections satisfying polynomial (1), with dates from a random number sample and all possible values for* x *and* y*.*

Figure 1: *Graph notations of the connections established by the polynomial (1).*

Expected Number of Connections

To investigate this question for the specific case of $a = 5,000$, $n = 77$ inspired by Velimir Khlebnikov, consider the set

$$\mathcal{D}_a = \left\{ P(x, y) : (x, y) \in \mathbb{Z}^2 \ \wedge \ |P(x, y)| \leq a \right\} \tag{3}$$

of possible, realizable differences between two numbers $m_i, m_j \in \mathcal{I}_a$. Note that $|365 + 48y| \geq 19$ for all $y \in \mathbb{Z}$ because $365 \equiv 29 \mod 48 \equiv -19 \mod 48$. Therefore, only those $x \in \mathbb{Z}$ contribute to differences in $\mathcal{D}_a$ that satisfy $|x| \leq \frac{a}{19}$. Furthermore, the difference $P(x, y)$ has to be smaller than a. Therefore, if $x > 0$, we obtain

$$y \leq \frac{a}{48y} - \frac{365}{48} \leq \frac{a - 365}{48}.$$

Equivalently, if $x < 0$, we obtain

$$y \geq \frac{-(a + 365)}{48}.$$

To enumerate the set $\mathcal{D}_a$, it suffices to iterate over the given ranges for x and y and to evaluate $P(x, y)$. In the specific case of $a = 5,000$, we have to iterate over $\{x \in \mathbb{Z} : -263 \leq x \leq 263\}$ and $\{y \in \mathbb{Z} : -111 \leq y \leq 96\}$. For these values, we obtain $|\mathcal{D}_{5,000}| = 2,479$.

If we pick two numbers m_1, m_2 from $\mathcal{I}_a$, the probability of these numbers to be connected is given as

$$\mathbb{P}\left(\text{pair connection in } \mathcal{I}_a^2\right) = \frac{\left|\left\{(m_i, m_j) \in \mathcal{I}_a^2 : m_i - m_j \in \mathcal{D}_a\right\}\right|}{(a + 1)^2}.$$

The reasoning is as follows. To count all connections in $\mathcal{I}_a^2$, we construct an $(a+1) \times (a+1)$ matrix. An entry (u, v) is equal to 1 if $u - v \in \mathcal{D}_a$. Otherwise the entry is equal to zero. Since $u - v \in \mathcal{D}_a \iff v - u \in \mathcal{D}_a$ the matrix is symmetric. Note that all entries on the diagonal are equal to 1 since we allow $x = 0$. Counting the ratio of 1 entries over the total number of matrix entries yields the probability of pair connections. Ordered pairs (m_i, m_j) for $m_i \neq m_j$ are double counted, both in the numerator and in the denominator. Again, in the specific case of $a = 5,000$, this probability is given as approximately 0.2309.

Finally, because of the linearity of the expectancy value, the expected number of connections is simply the number of pairs $\binom{n}{2}$ that can be found in the set $\{m_1, \ldots, m_n\}$ times the given probability of a pair to be connected. That is, in the case of $a = 5,000$ and for $n = 77$ chosen numbers, we expect a total of

$$\binom{77}{2} \cdot \mathbb{P}\left(\text{pair connection in } \mathcal{I}_a^2\right) \approx 675.8153$$

connected pairs. Note that the completed graph on Khlebnikov's data, as shown in Figure 1c has 690 edges and the randomly created graph in Figure 1d has 678 edges. Even if all of these connections are present in a subset of the graph, forming almost a clique, 38 dates would form this subset, i.e. about half of the given dates would be almost fully connected. This provides a mathematical explanation that shows the connections as defined by Velimir Khlebnikov to be expected and not at all relevant or special from a statistical point of view.

Density of $\mathcal{D}_a$ in $\mathcal{I}_a$

In his writings, Velimir Khlebnikov chose a specific time interval to establish his connections in. From a given number n of dates $m_1, \ldots, m_n$ in this interval, he extrapolates to other dates by establishing a connection.

Thereby, in the dialogue cited above, he extrapolates to the date 1917. However, when iterating over the whole interval and considering whether any date $m' \in \mathcal{I}_a$ is connected to some $m_1, \dots, m_n$, for the specific numbers of Khlebnikov, we find that all dates are connected to some m_i, see Figure 1c. That is, for any year in the considered interval, a connection via polynomial (1) can be established.

In the following, we will show an even stronger statement. Assume two dates are chosen, i.e. $n = 2$. Assume further, without loss of generality that $m_1 = 0$. In this case, any $m_2 \in \mathcal{I}_a$ is connected to m_1 if and only if $m_2 - m_1 \in \mathcal{D}_a$. In order to compute the probability of $m_2 \notin \mathcal{D}_a$, consider the set

$$\mathcal{D}'_a = \{d \in \mathcal{D}_a : \nexists d' \in \mathcal{D}_a, k \in \mathbb{N} \text{ s.t. } (|d'| < |d|) \wedge (d = d' \cdot k)\},$$

which is the set of all values in $\mathcal{D}_a$ such that no proper divisor of that value is also a value of $\mathcal{D}_a$.

Then, by convergence properties of infinite products as computed in calculus,

$$0 \leq \mathbb{P}(m_2 \notin \mathcal{D}_a) \approx \prod_{d \in \mathcal{D}'_a} \frac{d-1}{d} \xrightarrow{a \to \infty} 0 \qquad \Leftrightarrow \qquad \sum_{d \in \mathcal{D}'_a} \frac{1}{d} \xrightarrow{a \to \infty} \infty.$$

For a sufficiently large, the product approximates the probability arbitrary well. A (small) error in the approximation arises as a is not divisible by d in general and the formulation neglects the remainder.

Note that because of the definition of $\mathcal{D}'_a$, there exists no $d = P(x, y) \in \mathcal{D}'_a$ with $|x| > 1$. Therefore, a subset of the elements of $\mathcal{D}'_a$ is of the form $19 + 48y$. As 19 and 48 are co-prime, we can apply the strong form of Dirichlet's theorem [2, Sec. 3.3] which establishes that $\mathcal{D}'_a$ contains infinitely many primes and that the sum of their reciprocals is divergent.

Hence, for increasing a, the ratio of those numbers from $\mathcal{I}_a$ that lie in $\mathcal{D}_a$ over the total size of $\mathcal{I}_a$ goes to 1. Therefore, the expectancy of a random pair of numbers from $\mathcal{I}_a$ to be connected goes to 1 for increasing a. Note that this statement holds for n small in comparison to a.

Conclusion of the Mathematical Investigation and Artistic Outlook

The use of mathematical terminology and methodology provides any piece of writing or work with a scientific flair and a certain nimbus of authority. This effect has been observed for instance in the (mis-)use of statistics [7] or logical reasoning [3]. Clearly, a similar abuse of mathematics is present in the work of Velimir Khlebnikov when used for his obscure and impossible predictions of the future. In the sections above, we have shown how proper use of mathematical reasoning can debunk such claims. Namely, we have shown, that using the proposed methodology of Khlebnikov, the probability for a set of numbers to be completely connected goes to 1 with growing interval size a and comparably small number of dates n.

While the works of Velimir Khlebnikov remain a cornerstone of the Russian literature, they can still—despite their improper use of mathematics—be a benefit for the mathematical community. In the exhibition *Laws of Time. The future calculations by Velimir Khlebnikov* by video artist Kristina Paustian, the visitors were invited to explore the predictions obtained by polynomial (1) in a more personal manner than just for historical dates. On a website ([6], see Figure 2) that was on display on a computer in the exhibition, the user enters her or his year of birth. From a database, connections are constructed from the input to years of birth and years of death of famous mathematicians and composers. Thereby, the connection immediately becomes personally relevant. However, a repetition of the experiment yields more and new connections. Playing with the website reveals the sheer number of possible connections and thereby their arbitrariness. The poetic content and the playful website formed a low-threshold access to the underlying mathematics. Once the visitors to the exhibitions had played with the website and were puzzled by the number of connections obtained, a poster [4] next to the computer explained the underlying mathematics. Therefore, the exhibition served as a means for mathematical outreach.

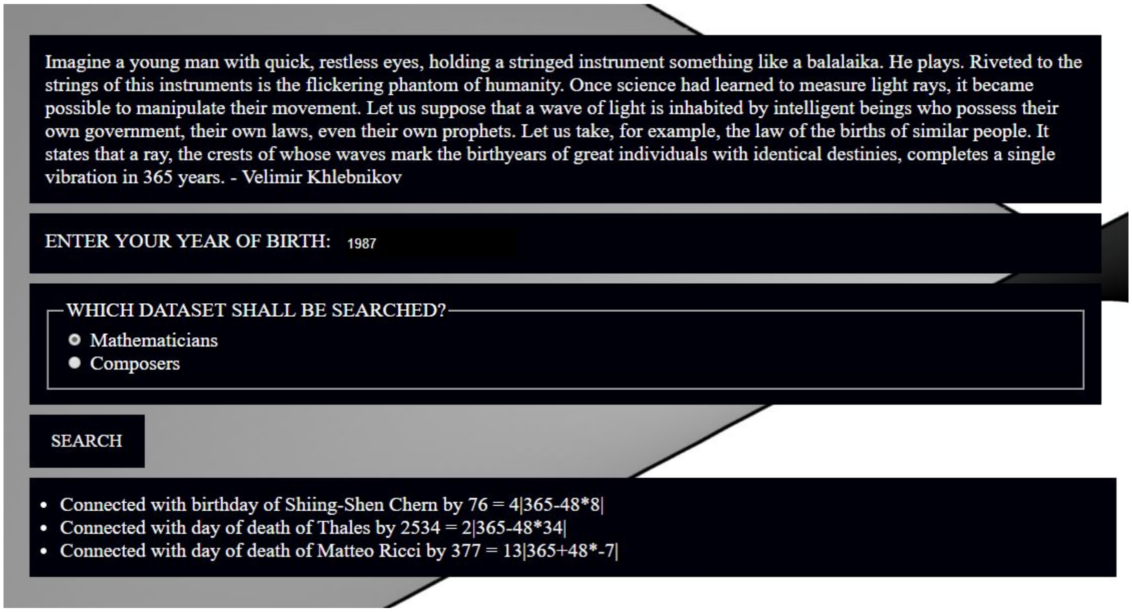


Figure 2: *Website "Laws of Similar Birth" on display at the exhibition by K. Paustian [6]. A user has entered a year of birth and the obtained connections are displayed.*

Artistic Exploration of Khlebnikov's Methodology

In the previous sections, we have presented mathematical reasoning why to reject Khlebnikov's methodology and his "laws of time". However, his construction gives rise to a two-dimensional surface embedded in $\mathbb{R}^3$. Furthermore, the relevant part of this surface is given by the set of integer points which are then used as distances in Khlebnikov's reasoning. In this final section, we present our own artistic approach to these mathematical structures in form of a digital artwork that does highlight some of the geometric properties of the surface not discussed so far.

Our rendering of the surface spanned by polynomial (1) is shown in Figure 3. We only show a small part of the infinite surface here. Following the reasoning from the above sections, the part shown includes exactly all those integer points $(x, y, P(x, y))$ with the property that $P(x, y) \in \{z \in \mathbb{Z} : -5000 \le z \le 5000\}$. Thereby, the displayed surface exhibits the complete set $\mathcal{D}_{5000}$ as defined in Equation (3) and used in the enumerative reasoning above.

To highlight the dual nature of the surface, it is split into two parts. The lower half—shown in black—contains the part of the surface where $P(x, y) < 0$. Integer point which lie in this half of the surface are marked by white dots. Vice versa, the upper half—shown in white—contains the part of the surface with $P(x, y) > 0$. Integer points lying in this half are given as black dots. As $x = 0$ is a valid input and yields $P(0, y) = 0$ for all $y \in \mathbb{Z}$, all points from the set $\{(0, z, 0) : z \in \mathbb{Z}\}$, i.e. all integer points on the y-axis, lie on the surface. These points are shown as gray dots.

Note that the dots are arranged on lines. These lines originate as for a fixed $\bar{y} \in \mathbb{Z}$, the polynomial $P(x, \bar{y}) = (365 + 48 \cdot \bar{y})$ is linear in x. Furthermore, it is obvious from this notation that all lines intersect at $(0, 0, 0)$. By this characterization, we see that $P(x, y)$ spans a ruled surface in $\mathbb{R}^3$. Furthermore, the two straight lines given by $x = 0$ and $y = -\frac{365}{48}$ are axes of two-fold rotational symmetry of the surface, where the first one maps black integer points to white integer points and vice versa.

We have seen in this paper that Khlebnikov uses mathematical notation rather as a Potemkin village. Scientific notation can be used to pursue goals that are on the contrary rather unscientific. Still, these concepts can give rise to interesting mathematical results and to aesthetic pieces of mathematical art as presented here.

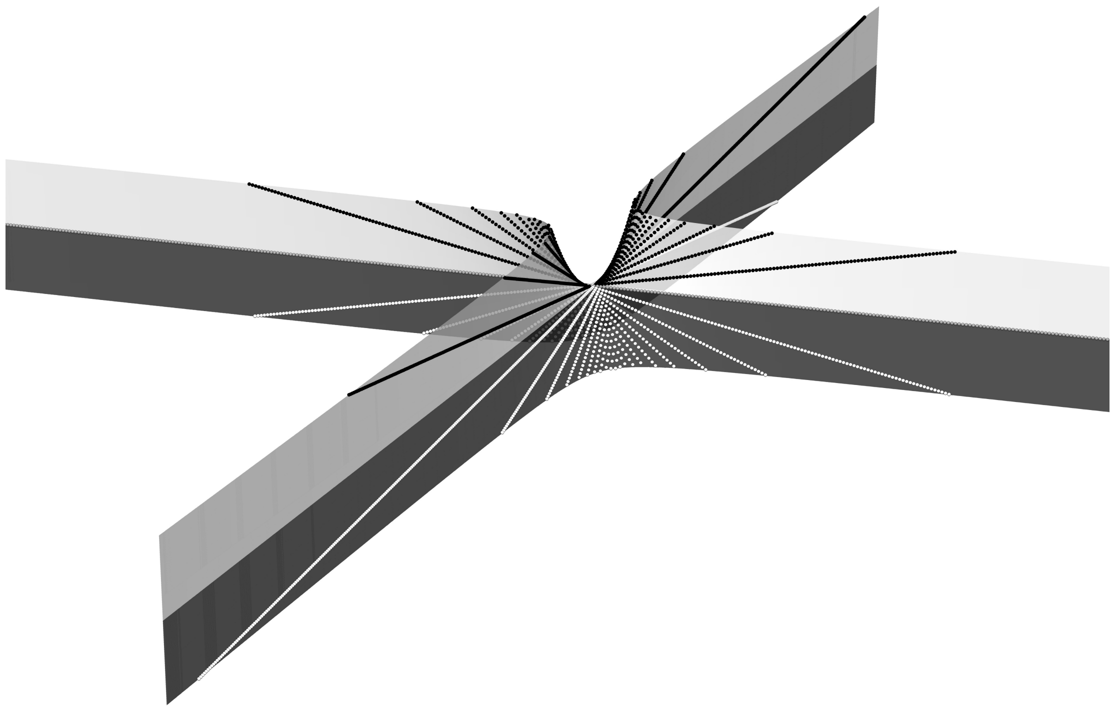


Figure 3: *An artistic rendering of the surface arising from Khlebnikov's polynomial (1). It is cropped to include the entire set $\mathcal{D}_{5000}$ as defined in Equation (3) and splits it into a negative and a positive part, shown in black and white respectively. The dots mark the corresponding integer points.*

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