

Science Communication and Outreach Events during the *Illustrating Mathematics* Semester Program at the *Institute for Computational and Experimental Research in Mathematics* (ICERM)

Good illustrations of mathematical content enhance the science communication of mathematics as well as the scientific process itself. However, the creation of these illustrations, the rendering of videos, the process of *mathematical making*¹, is still underdeveloped and undervalued by many professional mathematicians². The semester program *Illustrating Mathematics* aimed at closing some of the gaps between the richness of mathematical content and the availability of corresponding illustrations by creating new and novel ways to show and present mathematics.

This article reports on the program's related science communication and public outreach activities. To present the frame in which the activities took place, the first two sections briefly introduce the institutional setup and the scientific aspects of the semester program. The following sections each present one outreach activity in detail.

ICERM

In 2010, the five mathematicians Jill Pipher, Jeffrey Brock, Jan Hasthaven, Jeffrey Hoffstein, and Bjorn Sandstede founded the *Institute for Computational and Experimental Research in Mathematics* (ICERM)³. This was possible through financial support by Brown University, the National Science Foundation (NSF), and the Division of Mathematical Sciences. The goal of ICERM is to promote computational methods and experiments in mathematical research. It clearly says in the mission statement of ICERM:

"The mission of the *Institute for Computational and Experimental Research in Mathematics* (ICERM) is to support and broaden the relationship between mathematics and computation: specifically, to expand the use of computational and experimental methods in mathematics, support theoretical advances related to computation, and address problems posed by the existence and use of the computer through mathematical tools, research and innovation."⁴

ICERM executes this mission by hosting up to two semester programs (each with 3-4 affiliated workshops), several weeklong topical workshops, and a summer undergraduate research program every year. Aside from the board of directors, ICERM has a permanent staff of eleven people. They take care of, e.g., finance, available computer systems and software, as well as marketing and ICERM's internet presence. Except for at least one public lecture per semester, ICERM does not generally engage in outreach activities. Every summer the *Girls Get Math* event provides education for girls, but this event is not open to the public nor directed at a general audience. In 2019, ICERM was in its second 5-year NSF grant. These grants do not cover outreach or science communication activities. Still, the *Illustrating Mathematics* semester program was a welcome opportunity to engage in such events.

¹ Creation of illustrative content that is outside the traditional scope of visualization. It can comprise of, but is not limited to, literal objects, such as sculptures, paintings, or fabrics, but also digital images, software, or even performance arts.

² This is the underlying problem addressed in the *Mathematical Maker's Manifesto*. See: Farris, Frank. 2020. *Where does "mathematical making" fit in our community?* Notices of the American Mathematical Society 67 (5), 614–615, signed by 30 participants of the *Illustrating Mathematics* program.

³ Find more information about the institute on its website: <https://icerm.brown.edu/>.

⁴ Mission statement taken from <https://icerm.brown.edu/about/>.

The *Illustrating Mathematics* Program

From September to December 2019, the semester program *Illustrating Mathematics*⁵ took place at ICERM. This specific program brought together mathematicians, makers, and artists to find new forms of illustrations in and for mathematics.⁶ Counting all workshop attendees and members of the whole semester program, 257 people participated in the various program activities. Through a generous grant from the Alfred P. Sloan Foundation, this semester program was able to reach out into the local community of students as well as artists. Thus, the program got the opportunity to present its contents to a broader audience and not only to the visiting mathematicians. The program website states:

“Objects created for mathematical visualization are beautiful and attractive in their own right. ‘*Illustrating Mathematics*’ brings together artists, makers, and mathematicians seeking to harness the creativity of mathematical illustrations to further the public’s understanding of mathematical research. ICERM invites you to engage with these talented artists-in-residence and explore their work. Visualize mathematics through displays of art made using 3D printing, laser cutting, CNC routing, virtual reality, textiles, carving, painting, video, and more!”⁷

There were several events revolving around public outreach and mathematics communication during the program. They included an on-site and an off-site mathematical art exhibition, an ICERM open house during a STEM-themed city-wide *Waterfire*⁸ event, and five Math+Art panels featuring artist and mathematicians answering questions about how art and math relate from the audience. During *Waterfire*, and again later in the semester, the public were invited to participate in building two large mathematical sculptures in ICERM’s modern and open space. Diana Davis, one participant of the *Illustrating Mathematics* Program, edited a book, which collects the projects that the various participants created throughout the semester. The remainder of this article provides a detailed discussion of these events with a special focus on their science communication aspects.

Math+Art Exhibitions

Two exhibitions enabled the public to engage with art inspired by mathematics. Topics ranged from discrete and differential geometry over topology to combinatorics. The mathematical content is usually encoded in the art object and is not immediately accessible to the audience. Furthermore, the artist might reduce the underlying mathematical structure for either practical⁹ or artistic¹⁰ purposes. The exposition of the art did not include explanatory texts but only the title of the artwork and the name of the artist. The organizers chose this presentation style to create a feeling of a typical art exhibit. The mathematical background of the presented artworks is quite complex, generally not possible to understand from the artwork itself, and mostly requires more than a page of

⁵ Find a description of the program and its various activities here: <https://icerm.brown.edu/programs/sp-f19/>. Frank Farris describes a previous workshop with the same title in an article, which is available at <http://scitechconnect.elsevier.com/new-technology-mathematicians-turn-numbers-into-art/>.

⁶ This website contains a list of projects from the program: <https://im.icerm.brown.edu/>.

⁷ Public Outreach statement taken from <https://icerm.brown.edu/programs/sp-f19/#publicoutreach>.

⁸ The Waterfire is an event in Providence, RI taking place multiple times annually. It has attracted millions of visitors to come to downtown Providence, to see the burning fires in braziers on the water, and to interact with the local community. For more information, see <https://waterfire.org/>.

⁹ As an example for practical limitations, in the work *2-adic Solenoid* by Dina Buric, it is impossible to print the infinitude of small twisted fibers that form the solenoid. The gallery of accepted artworks lists the solenoid here: <http://gallery.bridgesmathart.org/exhibitions/2019-icerm-illustrating-mathematics/buricd>.

¹⁰ In her work *78 paths to decompose a sphere*, the artist Silviana Amethyst Brake cuts the ongoing paths around a sphere to focus on the center. Thereby, the actual mathematical paths outside the print are lost to the observer. Find the work here: <http://gallery.bridgesmathart.org/exhibitions/2019-icerm-illustrating-mathematics/danielleamethyst>.

introduction. Therefore, to prevent the conversion of the event from an art exhibit into a small-scale mathematical museum, the organizers only show the reduced information for each object.

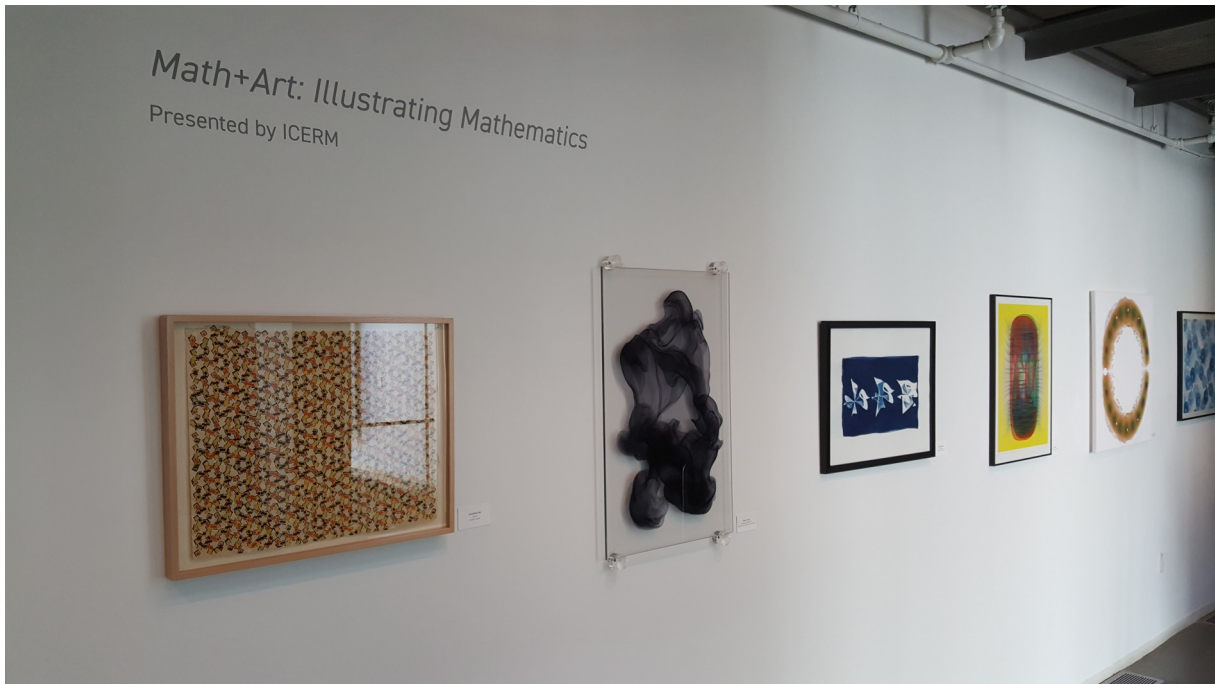


Figure 1 Part of the Exhibition at the Granoff Center. Credit: Mark. J. Stock.

Both exhibits ran parallel to the semester program. While one exhibition was located at ICERM from September to November 2019, the institute coordinated with the Brown Arts Initiative to host a second, more formal exhibit at the *Granoff Center*, on the campus of Brown University (see Figure 1). The Center serves as a place for teaching activities in the arts and as a hub for collaboration among the arts, sciences, and humanities.¹¹ Over the course of about four weeks, mostly students, faculty, and staff passed through the exhibition where the different art pieces confronted them with the underlying mathematics.

Two receptions at the *Granoff Center* enabled the interested public to engage with the artwork and the present artists. Despite open calls, the audience comprised almost completely of art students from the *Granoff Center*, artists from the local community, and members of the *Illustrating Mathematics* program. Smaller groups formed in front of the artworks and discussed the underlying mathematics, sometimes with the artists themselves, sometimes with other mathematicians present, and sometimes among themselves as laypersons. The exhibition at ICERM had similar effects: workshop visitors and visitors to the Math+Art Panels (see below for a separate discussion of these events) attended receptions taking place in the exhibition space (see Figure 2). Many visitors then wandered around and explored the exhibit, which gave the artists – if present – an opportunity to explain their artwork and subsequently also the underlying mathematics. The exhibition catalogue¹² printed at the end of the semester contains texts by the artists, describing their works and therefore presents a helpful tool to understand what mathematical theory inspired the artwork and how the artist chose to present this theory.

¹¹ Find more information on the *Granoff Center*, its mission, and its activities here: <https://arts.brown.edu/granoff-center>.

¹² *ICERM: Illustrating Mathematics Exhibition 2019*. David Bachman, Saul Schleimer, and Henry Segerman (Eds.). Tessellations Publishing, Phoenix, 2019.



Figure 2 Part of the exhibition at ICERM. Credit: Laura Taalman.

The organizers of the exhibits did not install any success measures or feedback mechanisms. In terms of challenges regarding the setup of the exhibitions, they report that these were only possible because of the on-line *Mathematical Art Galleries*¹³ infrastructure that many of the participating artists were familiar with. Setting up such infrastructure would have overstrained the locally available resources and thus would have rendered the entire project infeasible. The synergy of using an established curating system and not having to set up a new way of collecting submissions thus saved a tremendous amount of time and energy and therefore enabled the presentation of the mathematical art in the first place.¹⁴

Big Bang Science Fair during *Waterfire*

The *Waterfire*¹⁵ open house event enabled the public to engage with different mathematical (art) material. The activity at ICERM was part of the *Big Bang Science Fair*, which is an event that celebrates the intersections between science and the arts. The target group was both kids and adults who enjoy hands-on activities to discover science. At ICERM, the semester participants ran the event with their material and their prepared activities. Several tables represented different mathematical

¹³ The galleries started as a place to organize the annual art exhibit at the Bridges Conference (<https://bridgesmathart.org/>), but as of now also includes the exhibits at the Joint Mathematics Meetings (JMM, <https://jointmathematicsmeetings.org/jmm>) and two spin-offs of the Bridges Conference: a short film festival and a fashion show. Find the gallery devoted to the *Illustrating Mathematics* program here: <http://gallery.bridgesmathart.org/exhibitions/2019-icerm-illustrating-mathematics>.

¹⁴ The background information on ICERM, the *Illustrating Mathematics* program, and the two exhibitions come from the presented sources and from a personal conversation with J. Elisenda Grigsby, Deputy Director of ICERM.

¹⁵ See footnote 6 for a short description of the event.

topics: *two-dimensional geometries, three-dimensional geometries, mathematics in motion, and computational textiles.*

Each of the long tables held a multitude of mathematical illustrations and objects. The visitors were free to interact with all of these objects, i.e., lift them up, rearrange them, or use them to build objects that are more complex than their basic parts. Mathematicians behind the tables answered questions and explained the underlying mathematics of the presented exhibits. Children in particular showed great interest in the mostly colorful objects and in the possibilities of what to build from them (Figure 3).

Over the course of two hours, visitors interacted with the various objects and discussed with the available mathematicians. Aside from the four tables, the visitors could participate in a large-scale building activity to construct an illuminated *stellated dodecahedron* (Figure 4), organized by semester program participant Glen Whitney. These hands-on activities serve a twofold pedagogical purpose. First, the opportunity to build something themselves sparks the interest of children and adults alike. Second, it is easier to illustrate mathematical facts on some physical objects than just describing the mathematics. For instance, the visitors can easily count the number of vertices, edges, and faces of the large dodecahedral sculpture. From this information, it is possible to illustrate the Euler characteristic and thus give a simple example for a deep mathematical result. Furthermore, the illumination of the sculpture indicated the different symmetries encoded in the geometrical object.



Figure 3 Edmund Harris explains how to form complex object from basic Curvahedra tiles at the open house event during the Big Bang Science Fair. Credit: Ruth Crane.

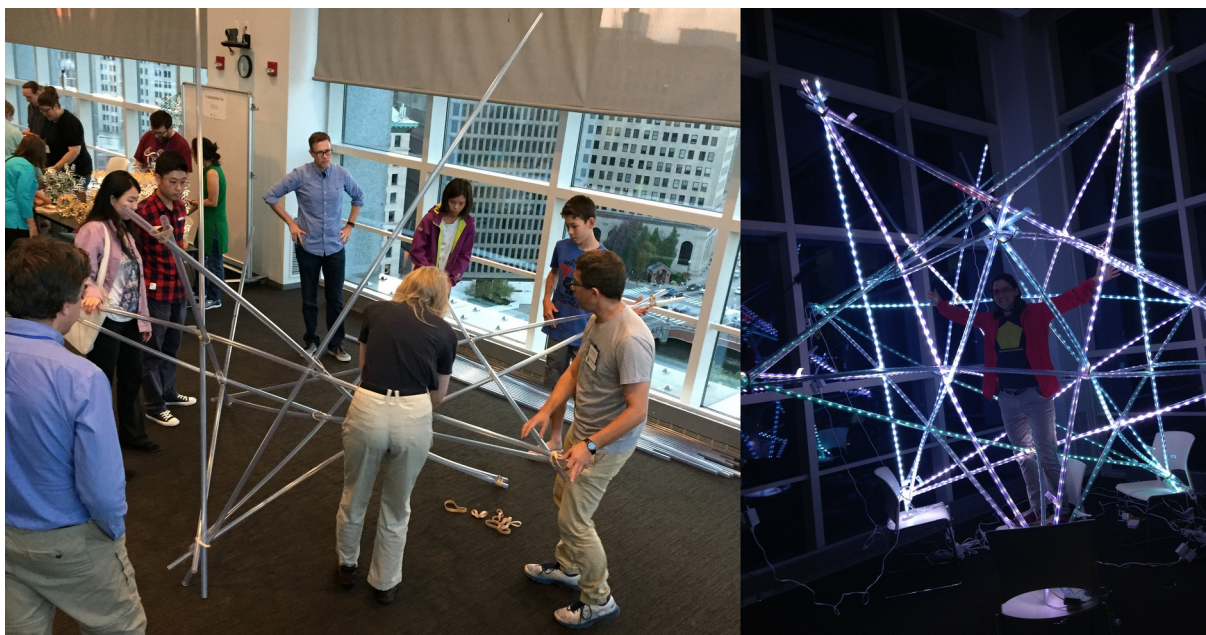


Figure 4 Left: Construction of the stellated dodecahedron at the open house event during the Big Bang Science Fair. Credit: Ruth Crane. Right: The finished and illuminated sculpture. Credit: Saul Schleimer.

A main goal of the participation of ICERM in the *Waterfire* event was raising awareness about the existence of the institute among the local residents of Providence and – on a larger scale – Rhode Island. While the local community knows Brown University as a whole, its different institutes are generally unknown. An event like the *Waterfire* is a great opportunity for smaller university entities – like ICERM – to make themselves known and to promote their research results. Towards this end, the event was a tremendous success for the institute. Because the semester-participants presented the materials they had produced so far or brought from home, preparation times were very short. The organizers of the *Big Bang Science Fair* advertised the event. In total, 300 free tickets to visit the ICERM open house were available and within hours, the distributors handed out all tickets. The public demand for more tickets would have been there, given the tenth of thousands of participants in the surrounding event, but the location could simply not host more attendees. However, even after the event ended, the mathematical sculpture continued to shine from the eleventh floor onto the visitors of the *Waterfire* event on the street below, thereby raising attention for ICERM.

Building Activity *Math's Bubbling (not!) Over*

Towards the end of the program, Glen Whitney organized another large-scale building activity open to the public in the conference room at ICERM titled *Math's Bubbling (not!) Over*. Throughout the course of a day, participants built a model of the *Weaire-Phelan (W-P) foam*¹⁶ from wooden dowels, custom plastic connectors, and acrylic cutouts for the faces of the foam cells. The organizer of the event writes on the goals of his activity:

“*Math's Bubbling (not!) Over* was originally planned to be built and displayed as a part of a public presentation about mathematics. In that context, the goal of the construction was to illustrate how discovering a mathematical pattern or phenomenon is rarely the end of a story, but typically leads to further layers of understanding and appreciation and new questions, in a potentially endless cyclic¹⁷ process. (I wanted to contrast this with the notion that a mathematician encounters a problem and then finds ‘the solution’ and then is done.)

¹⁶ The W-P foam is a hypothesized structure for an accumulation of a large (technically, infinite) number of bubbles of equal volume. Theoretically, such an accumulation would divide all of space into cells of equal volume with the minimum amount of surface area per cell. However, it remains unknown whether the *W-P foam* actually achieves the minimum possible surface area per cell.

Thus, confronted with a collection of line segments of specific lengths, the audience could discover that they could fit together to make certain pentagons and hexagons. Now equipped with those polygons, the audience could discover they created certain polyhedra, and then in turn that those polyhedra would fit together to tessellate all of space.¹⁸ Finally, the audience would be in a position to understand and appreciate the unsolved problem of whether this division of space is optimal in Kelvin's sense.¹⁹

However, the public presentation was not able to take place for personal reasons. Hence, the installation as it actually occurred became an opportunity for members of the ICERM community to converse in the context of a shared project, to experiment with construction techniques, and to observe the *W-P foam* at an unusual scale and from unusual viewpoints (e.g., from the interior of a cell). Based on feedback from participants, it seems that the installation succeeded in these altered goals.”



Figure 5 The fully constructed *W-P foam* model. Credit: Edmund Harris.

Aside from the goals of Glen Whitney, the building activity provided another means for ICERM to interact with the community and gain additional visibility. In terms of mathematical communication, it was an opportunity for members from the semester program to interact – given their own expertise – on a lesser-known subject of mathematics. Thereby, the building activity established communication between different sub-disciplines of mathematics.

¹⁷ While Glenn Whitney spoke about a “cyclic” process, it is safe to assume that he meant to describe it as a spiraling process that revisits certain questions while still going forward, unlike a cyclic repetition that always comes back to the exact same starting point.

¹⁸ That is to say that the entire space was covered without neither holes nor any overlap between the covering objects.

¹⁹ As stated in footnote 16, it is unknown whether the *W-P foam* achieves the minimum possible surface area per cell. Sir William Thomson (later Lord Kelvin) investigated this minimization problem in 1887.

Math+Art Panels

The origins of this activity originated through efforts of Rich E. Schwartz (mathematician at Brown University), Masha Ryskin (associate professor at the Rhode Island School of Design, RISD), and Allison Pashke (Rhode Island based artist). They aimed at involving RISD and the local art community in the semester program. ICERM and RISD are within walking distance but did not have any strong ties before this project. Schwartz frequently gives guest lectures on geometry in Ryskin's classes at RISD. Together with Allison Pashke, they came up with the idea to host panels at ICERM.

Throughout the *Illustrating Mathematics* semester, five panels were hosted, each of them scheduled for the duration of one hour. The members of a single panel consisted of two organizers and four participating artists and mathematicians. After a short introduction by the organizers, every participant had five minutes for a short presentation of her or his work. Discussions between the audience and the panelists followed these short talks and the panels usually went about 15 minutes over their one-hour time limit.

When preparing the panels, the organizers aimed for a variety of speakers. They abandoned a first idea to fix a topic for each panel and to invite corresponding panelists because it turned out to be logistically too complicated. Similarly, the organizers abandoned the idea of having exactly two artists and two mathematicians speaking in every panel. This is also because for most participants in the semester program, the organizers could not find good pairings of artists and mathematicians. The remaining goal was to provide a variety of speakers in each panel and to have at least one working mathematician speaking in each panel. The invitation process of speakers for the panel revealed a culture clash between the mathematics and the arts/design community. The two communities handle both the selection process and – even more important – the question of whether or not to pay the speakers differently. The second aspect of this conflict had to be resolved by finding funds to pay the speaking artists, which was possible by the grant of the Alfred P. Sloan Foundation. In regards of the first aspect, Masha Ryskin and Allison Pashke compiled a list of possible arts and design panelists, while Rich Schwartz compiled a list of possible mathematicians for the panel, thereby taking the varying selection criteria into account for both sets of speakers.

Early on during the planning, the organizers decided to overlap several of the panels with the workshops at ICERM. This had several benefits. On the logistics side, each workshop had a starting reception in the common area of ICERM. By scheduling the events right after these receptions, the interested public could engage the mathematicians and artists attending the workshop. Furthermore, the visitors had an opportunity to see the arts exhibition at ICERM (see above for a discussion of the exhibition). Rich Schwartz remarked that overlapping the panels with the workshop reception also gave a “classy air”²⁰ to the panels while the workshop participants provided a “fresh audience” to every panel. This contributed greatly to the discussions after the short presentations, driven by the audience. These discussions were different in every panel, ranging from very work-specific (“What problems did you encounter when cutting these wood pieces with a laser cutter?”) to rather broad and general questions (“When do you consider a piece ‘done’?”). See Figure 6 for a picture of one of the Math+Art panels.

Aside from the workshop participants, the organizers aimed their panels at interested students and faculty from both Brown University and RISD. It turned out to be very beneficial to have more than one panel as the event quickly established itself in the local art community and about a dozen local artists came to the last panel. Each panel had an audience of about one hundred people, which surpassed the expectations of the organizers who initially worried that not enough participants would come.

²⁰ Quotes from a personal conversation with Prof. Rich E. Schwartz, Brown University.



Figure 6 David Bachmann introduces the panelists of the second Math+Art panel. Credit: Ruth Crane.

In summary, the organizers acknowledge that the event would be hard to repeat without ICERM as a hosting institution. The logistics, support in handling speakers and invitations, rooms, catering, and paying of the artists by ICERM (with help from the Alfred P. Sloan Foundation) proved invaluable and crucial for the whole activity. In hindsight, the panels themselves went too quickly and there was not enough time to answer the questions in sufficient depth. An immediate success of the panel series was the start of several cooperative projects – both artistic and mathematical – between members of the audience and panelists. Furthermore, many students came to the panels and became interested in their contents, which Rich Schwartz sees as another big success. While he does not necessarily want to run another panel series, he surely wants to continue with other “lively math events for the public”.²¹

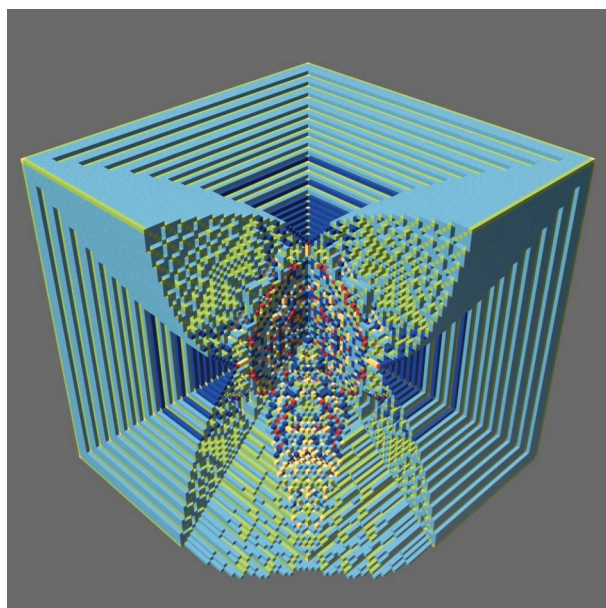
Of course it is not surprising that the co-organizer of the panels considers his event series a success. However, the sheer fact that the organizational efforts were too large to continue the series speaks for itself. Obviously, the gains do not compensate the efforts put into the organization. The questions show that the mathematical content was at most shallow, if present at all. Time did not permit for detailed discussions and while all of the images provided beautiful artwork inspired by mathematics, it has to be doubted that any actually mathematics were communicated during these events. Furthermore, while students and members of the local art community were presents, these were – for the most part – mathematics enthusiasts, such that the panelists preached to the choir. In this sense, the event did fail to reach out in the broader local community. Further evaluation and for instance a questionnaire handed out to the participants would shed some light on these aspects. Sadly, this opportunity is gone, but should be picked up in any similar event.

²¹ Quotes and background information on the Math+Art panels originate in a personal conversation with Prof. Rich E. Schwartz, Brown University.

The Illustrating Mathematics Book

A final outreach project that originated in the *Illustrating Mathematics* program is an eponymous book, edited by Diana Davis.²² Unlike an exhibition book that solely presents the objects and the artists, this book focuses on the medium used for the work as well as on the process of making the object. Therefore, it does not only include images of the final creations, but also stories from the process, like things that went wrong or got changed along the way. Guiding points for the authors were to elaborate on the medium, the mathematics on display, why the person chose this content, why they chose this medium to illustrate the content, what the creator learned from making the object, and what the observer can learn from the result.

The book accordingly has eight sections, each presenting works with a different medium. They consist of *Drawing, Paper & Fiber Arts, Laser Cutting, Graphics, Video & Virtual Reality, 3D Printing, and Mechanical Constructions & Other Materials*. The final section, *Multiple Ways to Illustrate the Same Thing*, explores how different media can capture different aspects of a given mathematical concept. The contributions highlight how different illustrations can complement and complete each other in the task of transporting the underlying mathematics. In the introduction, Diana Davis writes that her book does three things. It “showcases the great variety of materials for illustrating mathematics, gives voice to people’s stories about illustrating their mathematics, so that we can learn from their experience, and shows the variety of ways that different people use the same materials in very different ways.”²³



Chip firing is a process on a graph, which is a network of vertices connected by edges. Initially, a stack of “chips” is placed on each vertex. A vertex is allowed to “fire” its chips if it has at least one chip for each vertex to which it is connected (its “neighbors”). When a vertex fires, it sends one chip to each neighbor. We repeat this process until no vertex has enough chips to fire, which we call a “final configuration.”

The illustration shows chip firing on a cubical grid in 3D. Every vertex has six neighbors, so it needs six chips in order to fire. In this example, every vertex except for the origin started with 4 chips, and the origin started with 100,000 chips. The final configuration forms a cube. Vertices with 0, 1, 2, 3 or 4 chips are rendered as small cubes of different colors. Most of the vertices have 5 chips in the final configuration, so we rendered them as transparent, to exhibit the funnel-like shapes going into the cube from each of its faces. The octant of the cube facing the viewer is also transparent, and only half of the lower octant is shown. This is to show the inner structure of the final configuration, which resembles “leaves.”

A computer-generated rendering allows us to explore both the outer shape and the inner structure of the pattern interactively. Our original plan was to 3D print the patterns inside the object, but they turned out to be too fragmented, so a 3D print would fall apart. Having a digital model allowed for switching on and off the visibility of certain vertices, which made it easy to identify larger structures within the geometry.

Initially, we represented each vertex by its own cubical geometry object in the software Blender, but this yields too many objects, and crashed the Blender GUI. Therefore, we decided to create only six objects, each representing one of the numbers of remaining chips in the final configuration. Using the clipping plane built into the Blender camera, we could also clip the object and thus show any planar slice of the illustration. However, we wanted spherical, not planar clipping. Thus, we decided to build more geometries again: six geometries for each “shell” around the origin. That is, all cubes of a fixed radius distance to the origin and of the same color were collected in one object. By hiding the outside shells, it became possible to interactively show the internal layers of the structure.

Further information:
Caroline J. Klivans, *The Mathematics of Chip-Firing*, CRC Press (2018).



MARTIN SKRODZKI,
CAROLINE J. KLIVANS,
AND PEDRO F.
FELZENSZWALB
ICERM, Brown University,
Brown University
computer-generated graphic

Figure 7 A page from the *Illustrating Mathematics* book. It illustrates the process of three-dimensional chip firing. Image Credit: Caroline J. Klivans/Pedro F. Felzenszwalb/Martin Skrodzki. Credit: American Mathematical Society.

Figure 7 shows a representative page with the illustration work of the author of this article.²⁴ The illustrated mathematical content is the process of *chip firing* on an undirected graph network, sometimes also referred to as a *sandpile*. Starting with an initial stack of chips at a given starting position in the network, the process distributes these chips along the connections of the network. In each step, if possible, each network node distributes one chip to each of its connected neighbors. For a large enough graph, eventually, the process ends and forms intricate structures like the one shown in the left part of Figure 7. The book of Diana Davis collects 70 such illustration projects,

²² Davis, Diana (editor). *Illustrating Mathematics*. Providence, Rhode Island: American Mathematical Society.

²³ *Illustrating Mathematics*, p. 3.

²⁴ *Ibidem*, p. 62–63.

thereby serving not only as a chronicle of the *Illustrating Mathematics* program, but also as a source of inspiration for both further illustrations and upcoming illustration programs.

Final Thoughts and Outlook

While the activities presented in this article are the main events that took place during the *Illustrating Mathematics* semester at ICERM, there are more science communication projects to come. Mikael Johanson gave a talk on how to create Twitter Bots that present mathematical content automatically. One result is the bot of Dina Buric, which now posts daily fractal images based on the Tribonacci graph²⁵. Evelyn J. Lamb, a freelance math and science writer, gave a talk on mathematical writing for the public. The articles in her blog are great examples.²⁶ Steve Trettel and Sebastian Bozlee explored how to interact with mathematics in virtual reality.²⁷ These and more science communication projects originating from the *Illustrating Mathematics* semester at ICERM will be available in the coming months and years.

Acknowledgement

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²⁵ Find the bot of Dina Buric at <https://twitter.com/GraphTrib/>. Other mathematical twitter bots include tiling bot (<https://twitter.com/tilingbot>) by Roice Nelson and a bot on symmetric curves by Mikael Johanson (https://twitter.com/symmetric_curve), inspired by Frank Farris.

²⁶ <http://www.evelynjamb.com/>

²⁷ Sebastian's work is featured in several videos on YouTube, e.g., https://www.youtube.com/watch?v=N_bQ94bGHE0, while Steve's work can be interactively explored on his webpage <http://www.stevejtrettel.site/main/geo3D.html>.